

# Optimizing Rail Crane Mobility Through the Application of Non-Circular Bezier Curves

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**Abstract:** To address the issues of rail gnawing and rail jamming during the turning of a rail crane, a novel non-circular curve scheme based on Bezier curves is proposed. Specifically, a quasi-quartic Bezier curve characterized by three shape parameters is selected for the inner rail's turning path. The paths of the front and rear points on the crane's outer side are derived using the geometric relationships within the crane's traveling mechanism. An optimization objective is established, targeting the minimization of the deviation between these front and rear points. To achieve this, the optimal Bezier curve parameters are identified using a multi-start point heuristic global optimization algorithm. Subsequently, Hermite interpolation is employed to fit the outer rail trajectory. Computational results indicate a substantial reduction in the maximum deviation of the outer side points during the crane's turning when employing the quartic Bezier curve compared to traditional circular tracks. Furthermore, the deviation is further minimized by adjusting the three parameters of the quasi-quartic Bezier curve used for the inner rail.

**Keywords:** Rail crane, Bezier curve, deviation, global optimization algorithm.

## 1. Introduction

Multiple berths tend to cover modern large-scale ports, and the distribution of berths largely depends on the natural conditions on the shores of the ports. In order to realize the operation of a quayside container crane for multiple berths, the quayside container cranes need to be able to move between multiple berths. Usually, the crane only needs to run on the straight track, but some ports are not unidirectional to the sea, berths cannot be linearly distributed, so cranes need to turn and move. However, during the turning process of the rail-operated crane, there will be a deviation in the trajectory of the front and rear wheels, especially the multi-wheel operating system. If the deviation is small enough, it can be offset by the gap between the rim and the track, but if the deviation is too large, the gap is not enough to offset this deviation, which will cause dangerous situations such as gnawing and jamming between the wheel edge and the track, so the turning problem of the crane has increasingly become the focus of the industry with the development of the crane industry.

Bezier curve can smoothly transit with the straight line segment, and its continuity can be changed by adjusting the parameters of control points. In addition, in recent years, by introducing various Bernstein basis functions with parameters, the Bezier curve can achieve the effect of local adjustment without changing the position of the control points. Han and Liu

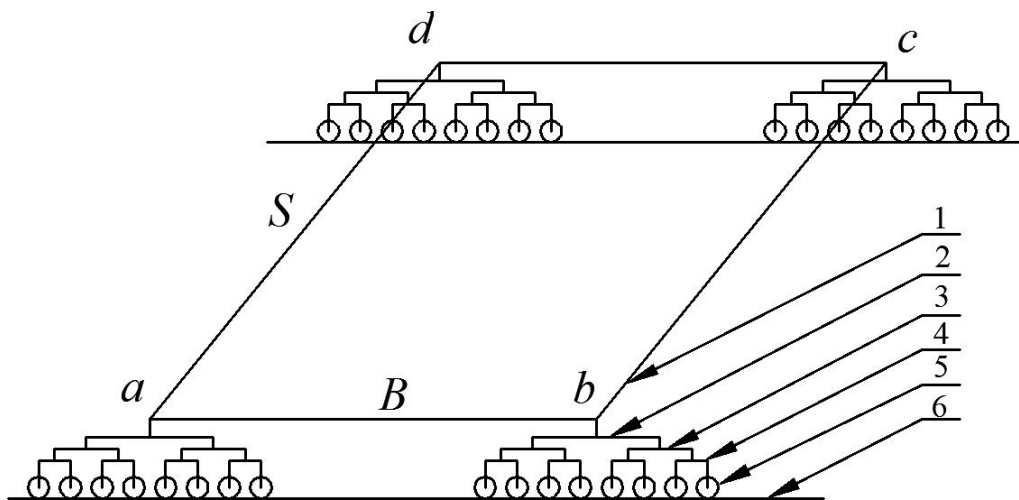
[1] introduced shape parameters into the basic function to extend the quartic Bezier curve. By changing the value of parameter in the numerical range, the degree of the curve close to the control polygon can be adjusted. In the extension of basis function of cubic Bezier curve, Hang et al. [2] introduced two parameters to make the curve more flexible in shape adjustability. Similarly, Liu et al. [3-5] also carried out corresponding research on the extension of the quartic Bezier curve, respectively introducing single, double and three parameters into the basis function. The new curve

constructed not only has similar characteristics with the quartic Bezier curve, but also has more flexibility and convenience in adjusting the shape.

According to the characteristics of the non-circular curve of crane turning, the quasi quartic Bezier curve with three shape parameters is introduced into the design of crane turning. Taking the deviation of the front and rear points of the outer rail as the objective function, the global optimization algorithm is adopted to optimize the parameters of the multiple control points to form the inner rail of the Bezier curve, and then the coordinate parameters of the front and rear points of the outer rail are deduced directly from the wheel coordinate parameter equation of the inner rail and the relevant parameters of the crane through the correlation of the rigid body motion. The coordinate points of the modified outer rail curve are determined according to the least squares method, etc. In order to ensure the smoothness of the curve obtained after calculation, it is necessary to smooth the discrete points and form the optimal outer rail by Hermite interpolation method. In this way, the optimization calculation and comparative analysis of the non-circular curved track of the single-wheel and multi-wheel situation of the crane are carried out, which provides a reference for the research on the design of the non-circular curved track of the crane.

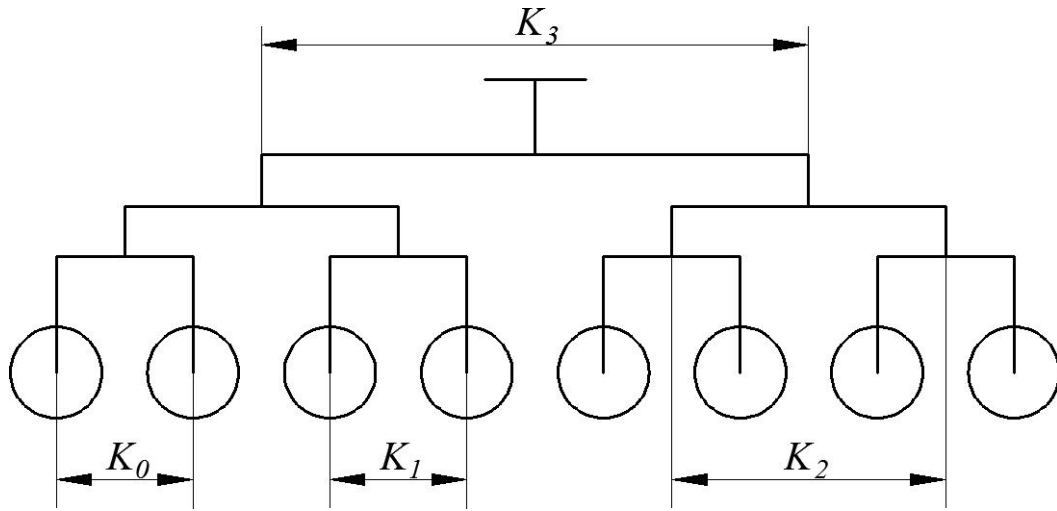
## 2. Principles of Rail Optimization

The existing crane carts generally adopt a four-point support structure, and the wheels are mostly double-rimmed cylindrical wheels. Figure 1 shows a simplified model of the traveling mechanism of the eight-wheeled cart, where  $B$  is the base distance of the crane and  $S$  is the gauge distance. Figure 2 shows the balanced beam span of the eight-wheeled cart, where  $K_0$  is the wheelbase,  $K_1$ ,  $K_2$ , and  $K_3$  are the spans of the first, second, and third-level balanced beam.



1. Simplified bottom frame of crane
2. Third-level balanced beam
3. Second-level balanced beam
4. First-level balanced beam
5. Wheel
6. Rail

**Figure 1.** Simplified model of eight-wheeled cart



**Figure 2.** Balanced beam span of the eight-wheeled cart

### 2.1. Quasi quartic Bezier Curve with Three Shape Parameters

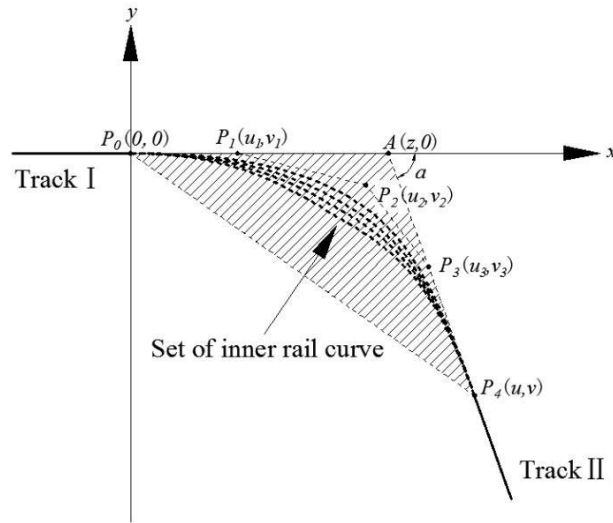
$$\begin{cases} b_{0,4}(t) = (1 - \lambda t)(1 - t)^4 \\ b_{1,4}(t) = (4 + \lambda - \lambda t)(1 - t)^3 t \\ b_{2,4}(t) = (6 - \beta t)(1 - t)^2 t^2 \\ b_{3,4}(t) = [4 + \alpha t + \beta(1 - t)](1 - t)t^3 \\ b_{4,4}(t) = (1 - \alpha + \alpha t)t^4 \end{cases}$$

Definition 1, Polynomial about  $t$ :

### 2.2. Method of Inner Rail Forming

As shown in Figure 3, the quasi quartic Bezier curve has five control points and three parameters. In order to ensure the tangency between the curved track and the straight track, the initial and final fixed control points  $P_0$  and  $P_4$  are set at the junction of the two straight tracks and the curved track. The two straight tracks intersect at point  $A(z, 0)$ , and the angle between the straight track II and the horizontal axis is  $\alpha$ . The variable control points  $P_1, P_3$  are located on the extension line of the two straight tracks, and  $P_2$  is located in the triangular area surrounded by the two straight tracks. By adjusting the positions of the control points  $P_1$  and  $P_3$  on the line segments  $AP_0$  and  $AP_4$  and the position of the  $P_2$  in the triangular area, the set of inner rail curve can be obtained. The deviation of the front and rear points of the outer rail corresponding to each curve can be obtained according to the theoretical calculation.

Finally, the curve corresponding to the minimum deviation can be selected as the inner rail during the turning process of the crane.



**Figure 3.** Inner rail based on quasi quartic Bezier curve

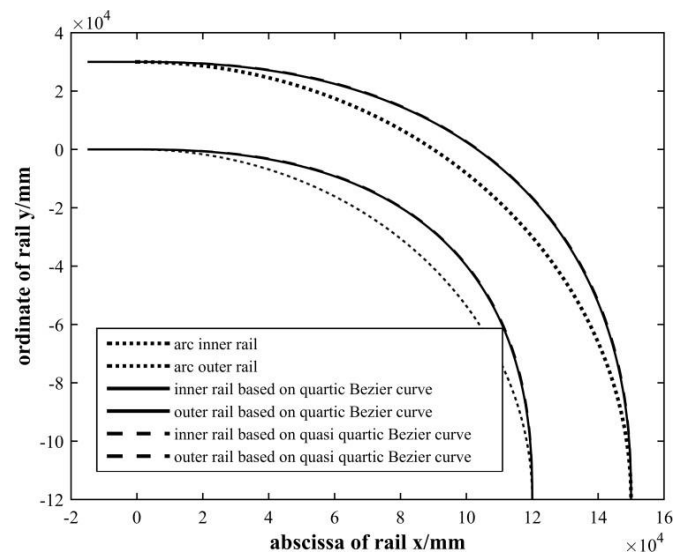
Bring the horizontal and vertical coordinate values into formula (2):

$$\begin{cases} x = (4 + \lambda - \lambda t)(1-t)^3 t u_1 + (6 - \beta t)(1-t)^2 t^2 u_2 + [4 + \alpha t + \beta(1-t)](1-t)t^3 u_3 + (1 - \alpha + \alpha t)t^4 u \\ y = (6 - \beta t)(1-t)^2 t^2 v_2 + [4 + \alpha t + \beta(1-t)](1-t)t^3 v_3 + (1 - \alpha + \alpha t)t^4 v \end{cases}$$

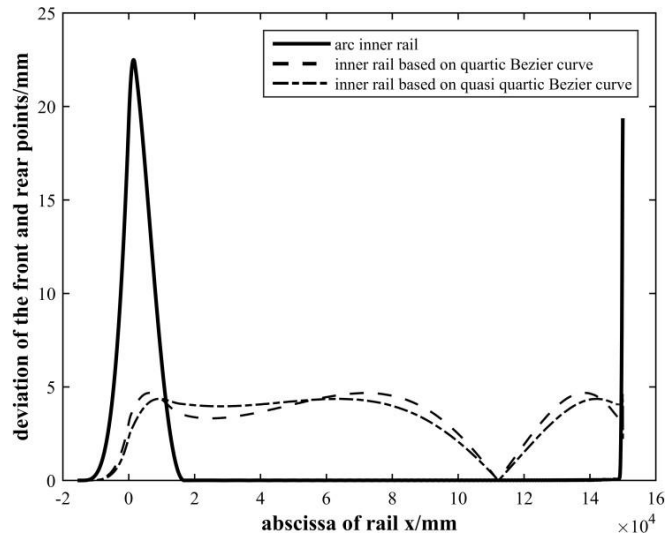
### 3. Case Calculation and Analysis

According to the model of crane turning established above, by selecting the track angle  $\alpha=90^\circ$ , the gauge distance  $S=30$  m, the base distance  $B=15$  m, the track  $K_0=1$  m, the radius of arc inner rail  $R=120$  m and calculating by global optimization algorithm.

Under the same turning parameters, the comparison of the inner and outer rail trajectories and the deviations of outer front and rear points between the circular rail, quartic Bezier curve track and quasi quartic Bezier curve track are shown in Figure 4.



(a) Comparison of trajectories



(b) Comparison of the deviations of front and rear points

**Figure 4.** Comparison of trajectories and the deviations of front and rear points

When the crane runs on the circular rail, the maximum deviation is 22.49 mm at the just- entering of the curve, and the deviation is zero after entering the curve completely, which means the front and rear trajectories coincide and no deviation problem occurs, and a larger deviation occurs again when the crane just leaves the curve. When the Bezier curve is used as the inner rail, the maximum deviation will not exceed 4.7 mm, and the deviation of trajectory is very small and smooth during the turning process, especially avoiding the sharp increase of the deviation when the cart enters and leaves the curve, thus greatly reducing the probability of jamming and gnawing when the cart just enters and leaves the curve, and highlighting the superiority of the Bezier curve as the inner rail.

After further observation of Figure 4, it is found that the inner and outer trajectories are almost coincident when using quartic Bezier curve and quasi quartic Bezier curve with three shape parameters as inner rails, and their shape is only slightly different in the local position, but there are obvious differences in the deviations. When the quasi quartic Bezier curve is chosen as the inner rail, the maximum deviation is 4.3645 mm, which is not only smaller than the maximum deviation which is 4.6819 mm when the quartic Bezier curve is used as the inner rail, but also the overall trend of the deviation is more stable, which guarantees the traceability of the front and rear wheels on the outside. Therefore, it will achieve better results in avoiding and eliminating the hidden dangers that wheels gnaw and jam rails.

## 4. Conclusion

(1) In the rail optimization of non-circular curve of crane turning, the Bezier curve is introduced in this paper. Taking the minimum deviation of the outer front and back points as the optimized target, the global optimization algorithm is used to search for the best inner rail. Finally, the trajectory of outer rail is fitted by interpolation method. By comparing the quartic Bezier curve as the inner rail with the traditional circular rail, the results show that the deviation of the circular inner rail is several times larger than that of the inner rail based on quartic Bezier curve when the crane just enters and leaves the curve, which is also an important reason for crane jamming and gnawing rails. This highlights the advantage of the Bezier curve as the inner rail of crane turning.

(2) In order to further reduce the deviation, a quasi quartic Bezier curve with three shape parameters is introduced in this paper. Similarly, the algorithm is used to find the optimal position of the control points and the optimal value of three parameters which form the best inner rail. The results show that the three shape parameters have different effects on the inner rail, and the inner rail can be fine-tuned by changing the value of the parameters. Therefore, compared with the quartic Bezier curve, using the quasi quartic Bezier curve with three shape parameters as the inner rail not

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only further reduces the maximum deviation, but also makes the deviation of the front and rear points of the outer rail tend to be stable, which guarantees the traceability of the outer front and rear wheels in the course of traveling and reduces the probability of that wheels jam and gnaw rails.

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